

# Spacetime structure of the global vortex

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## Abstract

We analyse the spacetime structure of the global vortex and its maximal analytic extension in an arbitrary number of spacetime dimensions. We find that the vortex compactifies space on the scale of the Hubble expansion of its worldvolume, in a manner reminiscent of that of the domain wall. We calculate the effective volume of this compactification and remark on its relevance to hierarchy resolution with extra dimensions. We also consider strongly gravitating vortices and derive bounds on the existence of a global vortex solution.

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## I. INTRODUCTION

The global vortex is a topologically non-trivial vacuum defect solution in  $2 + 1$  dimensions (or  $2 + 1 + p$  dimensions, in which case the vortex is a  $p$ -brane). Derrick's theorem would normally lead us to expect that there could be no such soliton, however, the vortex evades Derrick's theorem by having not a finite energy, but a logarithmically divergent one. This divergence naturally means that when the gravitational interactions of the vortex are included, the story becomes rather interesting. At first, it was assumed that the spacetime of a global vortex would be static, like its local Nielsen-Olesen cousin, however, such an assumption led to an exact, but singular metric [1] outside the core of the vortex. It was rapidly realised that any static global vortex (including those derived from a sigma model) would be singular [2,3].

The global vortex is however, one of a family of global defect solutions. The domain wall, which separates regions of discrete vacua, is perhaps the simplest, however, there is also the global monopole [4] which has a linearly divergent energy and is unstable [5]. Of the three global defects (in four dimensions) only the wall has a finite energy, indeed in more than four dimensions where there are further global  $p$ -brane solutions analogous to the monopole [6], the wall continues to be the only defect with a finite energy (per unit brane volume). Curiously this lack of finiteness of monopole energy does not lead to gravitational singularities as it does for the vortex, for the global monopole has a well-defined static gravitational field [4] (as does its higher dimensional descendents [6]) which is an asymptotically locally flat (ALF) spacetime with a global conical deficit angle.

The gravitational field of the domain wall is well studied (e.g. [7–10]), and from the perspective of an observer on the wall is non-static, having a Hubble expansion parallel to the wall. This however is known to be illusory, in the sense that it is the wall itself which is in motion, in a static Minkowski spacetime [9]. The vacuum domain wall is in fact an accelerating bubble, which contracts in from infinite radius, decelerating until it reaches a minimum size, then re-expanding out again to future null infinity. Constant time surfaces are then topologically spherical, and the wall can be viewed as self-compactifying spacetime on a scale roughly of its inverse energy. This picture of course refers to an infinitesimally thin wall, however, the thick topological defect solution has the same geometry, with the scalar field simply following the contours of the bubble [10].

In the broader context of the global defect family, the singular nature of the static global vortex spacetime was anomalous. The resolution of the puzzle was of course to add time dependence to the vortex metric [11], which allowed for the existence of a non-singular solution to be demonstrated. This time dependence was of the same nature as that of the vacuum domain wall spacetime, *i.e.*, a Hubble expansion along the length of the vortex, however, the full global spacetime structure of the global vortex and its maximal analytic extension was not explored, and indeed there have been claims that the cosmological event horizon present in the vortex spacetime is unstable to perturbations and becomes singular [12].

More recently the issue of global defect solutions has acquired a new dimension (quite literally!) as the renaissance of the braneworld scenario has resulted in a search for new exotic compactifications. The braneworld scenario is a compelling picture in which instead of having a Kaluza-Klein compactification of extra dimensions, we have relatively large extra

dimensions, but are confined to a four-dimensional submanifold within this larger spacetime. The Randall-Sundrum (RS) scenario [13] is an example of a particular warped compactification which has many features in common with the more ‘realistic’ string-motivated heterotic M-compactifications [14]. The RS scenario consists of one or two reflection symmetric domain walls at the edge of an anti de Sitter bulk, with only gravity propagating in the bulk. Naturally a wall can only ever give a warped compactification with one extra dimension, and it is interesting to explore warped compactifications of higher codimension.

Warped compactifications with local defects have been explored in a variety of papers [15], however, these compactifications are often either asymptotically flat in the extra dimensions, or have singularities. The situation with global defect compactifications is slightly different. In vacuum, a static vortex compactification is singular [16], although analogous to the four dimensional global string, this singularity can be removed either by the addition of time dependence (see the next section), or by adding a negative bulk cosmological constant [17]. Higher co-dimension global defect spacetimes based on the global monopole were considered in [6], and analogous to the global monopole tend to exhibit global deficit angles.

The question we are interested in in this paper is the spacetime structure of the global vortex. The presence of an event horizon, as well as the Hubble expansion along the vortex, indicates that something similar to the domain wall spacetime might in fact occur. In [18] an extension of the vortex spacetime was proposed which glued together a vortex and anti-vortex ( $V\bar{V}$ ) space to form a sphere with the vortex and anti-vortex at antipodal points. It was pointed out in [6] that this solution could have an alternate interpretation as a vortex around an equatorial circle of an  $S^3$ , just as the higher dimensional global  $p$ -branes can sit on big circles of de-Sitter spheres. However, these two situations are somewhat different; the global  $p$ -brane has an ALF spacetime, so adding a global  $p$ -brane to the already compact de Sitter space is simply a question of embedding. The vortex on the other hand has a strong effect on spacetime far from its core, and so if it lies on any great circle of a compact sphere, then it can only do so by actually self-compactifying spacetime. In this paper, we show that this is indeed what happens, by considering a generalization of the coordinate transformation used in [11] to explore the cosmological event horizon (CEH). We also show how to extend the metric beyond the CEH, and derive the maximal analytic extension of the global vortex spacetime.

## II. THE GLOBAL VORTEX PHASE PLANE

In this section we review and extend the arguments of [11,17] for the existence of a global vortex solution in  $p+3$  dimensions. We will be looking for a topologically nontrivial solution of the field theoretic lagrangian

$$\mathcal{L} = (\nabla_\mu \Phi)^\dagger \nabla^\mu \Phi - \frac{\lambda}{4} (\Phi^\dagger \Phi - \eta^2)^2 \quad (1)$$

in an otherwise empty spacetime. By writing

$$\Phi = \eta X e^{i\chi} \quad (2)$$

we can reformulate the complex scalar field into two real interacting scalar fields, one of which ( $X$ ) is massive, the other ( $\chi$ ) being the massless Goldstone boson responsible for the

divergent energy of the vortex. In this way, the low energy theory is seen to be equivalent in  $n(=p+3)$  dimensions to an  $(n-2)$ -form potential. For example in four dimensions, the  $\chi$ -field is equivalent to a Kalb-Ramond  $B_{\mu\nu}$  field, and the effective action for the motion of a global string is the bosonic part of the superstring action.

A vortex solution is characterised by the existence of closed loops in space for which the phase of  $\Phi$  winds around  $\Phi = 0$  as a closed loop is traversed. This in turn implies that  $\Phi$  itself has a zero within that loop, and this is the core of the vortex. From now on, we shall look for a solution describing a vortex with unit winding number *i.e.*,  $\chi = \theta$ , where  $\partial_\theta$  is some Killing vector of spacetime with closed circular orbits.

The boost symmetry of the energy momentum tensor parallel to the defect means that the most general metric can be written in the form

$$ds^2 = e^{2A(r)} H^{-2} g_{\mu\nu} dx^\mu dx^\nu - dr^2 - C^2(r) d\theta^2 \quad (3)$$

where  $\mu, \nu = 0, 1 \dots p$  are the coordinates parallel to the defect,  $g_{\mu\nu}$  is a constant unit-curvature de Sitter metric, and  $H$  is a constant to be determined which will represent the Hubble parameter of the expansion on the vortex braneworld.

The Einstein equations for the vortex are found to be

$$\left[ e^{(p+1)A} C' \right]' = -\epsilon e^{(p+1)A} \left( \frac{2X^2}{C} + \frac{C(X^2 - 1)^2}{2(p+1)} \right) \quad (4a)$$

$$\left[ C e^{(p+1)A} A' \right]' - C H^2 p e^{(p-1)A} = -\epsilon e^{(p+1)A} \left( \frac{C(X^2 - 1)^2}{2(p+1)} \right) \quad (4b)$$

$$(p+1) \left[ \frac{p}{2} (H^2 e^{-2A} - A'^2) - \frac{A' C'}{C} \right] = -\epsilon X'^2 + \frac{\epsilon X^2}{C^2} + \frac{\epsilon}{4} (X^2 - 1)^2 \quad (4c)$$

$$\left[ C e^{(p+1)A} X' \right]' = C e^{(p+1)A} \left[ \frac{X}{C^2} + \frac{1}{2} X (X^2 - 1) \right] \quad (4d)$$

where we have taken  $\lambda\eta^2 = 1$ , and  $\epsilon = \frac{r^2}{2} (M_n)^{-(p+1)}$  which represents the gravitational strength of the vortex.

In what follows, unless stated otherwise, we will take the vortex to be weakly (or at least relatively weakly) gravitating, *i.e.*,  $\epsilon \ll 1$ ; we comment on large  $\epsilon$  in section IV. We can then see that  $H$  is at least  $O(\epsilon)$  and noting that to leading order (4d) implies

$$\left[ X^2 - r^2 X'^2 + \frac{r^2}{4} (X^2 - 1)^2 \right]' = \frac{r}{2} (X^2 - 1)^2, \quad (5)$$

we see that outside the core

$$e^{(p+1)A} = 1 - \frac{\epsilon}{2} - \epsilon \ln \left( \frac{r}{r_c} \right) + \epsilon \int_0^{r_c} r X'^2 + O(\epsilon^2) \quad (6a)$$

$$C' = 1 - \epsilon\mu + \frac{\epsilon p}{(p+1)} - \epsilon \ln \left( \frac{r}{r_c} \right) + O(\epsilon^2) \quad (6b)$$

where  $\mu$  is the “renormalized” energy per unit length (*i.e.*, ignoring the logarithmically divergent term)

$$\mu = \int_0^{r_c} \left[ r X'^2 + \frac{X^2}{r} + \frac{r}{4} (X^2 - 1)^2 \right] , \quad (7)$$

and  $r_c$  is an effective width of the vortex, of order unity, outside of which  $X \simeq 1$ . These linearized forms can be used outside the core while  $r \ll r_c e^{1/\epsilon}$ .

In [11] it was shown that a non-singular solution had to asymptote an event horizon, *i.e.*,

$$e^A \simeq H(r_H - r) \quad , \quad C \simeq C_0(1 + O(r_H - r)^2) \quad (8)$$

as  $r \rightarrow r_H$ , the horizon radius. It is not difficult to show that this remains the case for general  $p$ . A quick integration of (4b) then indicates that  $H^2 = O(e^{-1/\epsilon})$ , a rough estimate which will be borne out by the more detailed calculations later in this section.

In order to demonstrate the existence of the vortex, we therefore need to verify the existence of a solution interpolating between the core solution (6a,6b) and the horizon. To do this, we examine the field equations outside the core, *i.e.*, we set  $X \simeq 1$ . Writing  $\rho = \int e^{-A} dr$ , and defining

$$x = p \frac{dA}{d\rho} + \frac{1}{C} \frac{dC}{d\rho} \quad ; \quad y = \frac{1}{C} \frac{dC}{d\rho} \quad (9)$$

these field equations can be written as a two dimensional dynamical system

$$\frac{dx}{d\rho} = \frac{x^2}{p} - \frac{(p+1)}{p} y^2 - p H^2 \quad (10a)$$

$$\frac{dy}{d\rho} = \frac{(p+1)}{p} x^2 - \frac{(p+1)}{p} y^2 - p(p+1) H^2 - xy \quad (10b)$$

with the constraint

$$\frac{2\epsilon p e^{2A}}{(p+1)C^2} = - (x^2 - y^2 - p^2 H^2) \quad (11)$$

The phase plane for this system is characterized by the critical points

$$P_{\pm} = H (\pm p, 0) \quad (12a)$$

$$Q_{\pm} = H \left( \pm \sqrt{p(p+1)}, \pm \sqrt{\frac{p}{p+1}} \right) \quad (12b)$$

and the invariant hyperboloid

$$x^2 - y^2 = p^2 H^2 . \quad (13)$$

The critical points  $P_{\pm}$  are saddle points while the classification for  $Q_{\pm}$  depends on the value of  $p$ : for  $p < 7$ ,  $Q_+/Q_-$  is a stable/unstable focus while for  $p \geq 7$ ,  $Q_+$  is an attractor and  $Q_-$  a repeller. Even so, the qualitative shape of the phase plane remains approximately the same irrespective of the value of  $p$ , provided that one re-scales the coordinates  $x$  and  $y$ . The phase space is represented in figure 1 where we have chosen  $p = 3$ .

The asymptotic solutions corresponding to the critical points are:

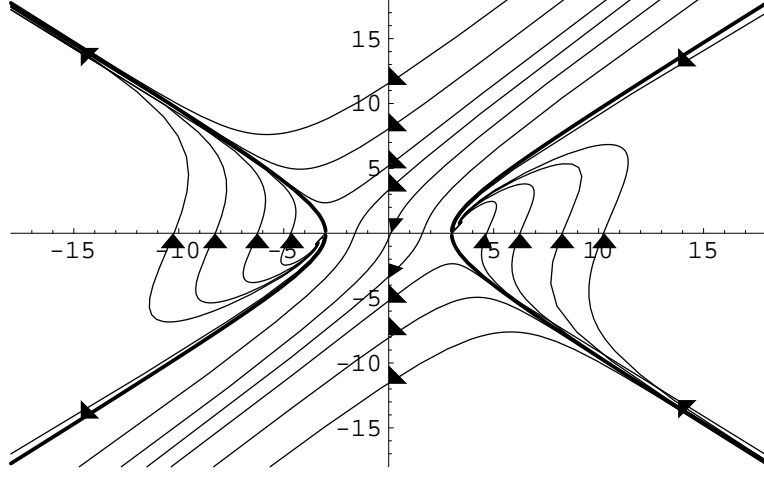


FIG. 1. The phase space  $y(x)$  for the global vortex 3-brane. The thick line represents the invariant hyperboloid in eqn (13) and the critical points are located at  $P_{\pm} = (\pm 3, 0)$  and  $Q_{\pm} = \pm(2\sqrt{3}, \frac{\sqrt{3}}{2})$  where we have fixed  $H = 1$  through a redefinition of  $\rho$ .

$$P_{\pm} : e^{A(r)} \simeq \pm H (r - r_H), \quad C(r) \simeq C_0 \left[ 1 - \frac{\epsilon(r - r_H)^2}{(p+2)C_0^2} \right], \quad r \rightarrow r_H^{\pm} \quad (14a)$$

$$Q_{\pm} : e^{A(r)} \simeq \pm H r \sqrt{\frac{p}{p+1}}, \quad C(r) \simeq \pm r \sqrt{\frac{2\epsilon}{p+1}}, \quad r \rightarrow \pm\infty \quad (14b)$$

where  $r_H$  is the horizon of the defect in the transversal section of the brane. Note however that the solution for  $Q_{\pm}$  should be treated with some caution, as this lies outside the physically allowed region of the phase plane as defined by positivity of the constraint (11). However, we will see in the next section that  $Q_{\pm}$  is relevant for the analytically continued vortex spacetime outside the CEH.

Now consider the critical point  $P_-$ . From the constraint (11) we see that this corresponds to the vortex horizon as  $\frac{dC}{d\rho} = e^{2A} = 0$  and  $\frac{dA}{d\rho} = -H$ . Therefore a non-singular solution exists for the vortex  $p$ -brane if a trajectory which corresponds to the initial conditions emerging from the core at  $r_c$  approaches this critical point. Note that  $H$  is as yet undetermined. From (10a) and (10b) we see that rescaling  $H$  simply rescales the phase plane. We therefore determine  $H$  by the requirement that the nonsingular trajectory must pass through the initial  $(x_c, y_c)$  determined from the core solution (6a,6b). This is what we now investigate. We separate the problem into two parts: the behaviour of the trajectories from the core to the  $y$ -axis, and then from the  $y$ -axis to the critical point.

First however, we make some qualitative remarks about the phase plane trajectories. Using the constraint (11) we can rewrite the dynamical system (10a)-(10b) as

$$\frac{dx}{d\rho} = -\frac{2\epsilon e^{2A}}{C^2(p+1)} - y^2 \quad (15a)$$

$$\frac{dy}{d\rho} = -\frac{2\epsilon e^{2A}}{C^2} - x y \quad (15b)$$

which shows that  $x$  is monotonically decreasing along a trajectory, whereas  $y$  is decreasing

only when  $y$  and  $x$  are both positive. Thus the trajectories coming from the core (which have  $y \simeq x = O(1)$ ) cross the  $y$ -axis. Once they have crossed to  $x < 0$ , either  $y(x=0)$  is sufficiently large that the trajectories have a turning point and pass to increasing  $y$ , or they hit  $y = 0$ . The critical nonsingular trajectory of course asymptotes  $y = 0$  at  $P_-$ , however the others all pass to  $x, y \rightarrow -\infty$ . This shows why it is important to analyse the crossing of the  $y$ -axis.

In order to characterize the trajectories near the core we use (6a) and (6b) to see that

$$A' \simeq \frac{dA}{d\rho} = \frac{-\epsilon}{(p+1)r_c} + O(\epsilon^2) \quad (16)$$

and hence for trajectories near the core

$$\frac{y}{x} = \left(1 + \frac{pA'C}{C'}\right)^{-1} = 1 + \frac{p\epsilon}{(p+1)} + O(\epsilon^2) \quad (17)$$

with

$$y_c = \frac{1}{r_c} \quad (18)$$

from (11). These give the initial conditions for the integration of the dynamical system at the ‘edge’ of the vortex,  $r_c$ . At next to leading order one gets

$$y = x \left[1 + \frac{p\epsilon}{(p+1)(1 + \epsilon \ln(x/x_c))}\right]. \quad (19)$$

Indeed, while  $x, y \gg pH$ , we may ignore the  $H^2$  terms in (10a,10b) which reduces our system to the Cohen-Kaplan (CK) analysis. Hence, writing  $u = u_c - \int_{r_c}^r C dr$  (so that  $u = 0$  corresponds to the CK singularity and  $u = u_c \simeq 1/\epsilon$  the edge of the vortex – see [16]) our solution in this régime is

$$e^{(p+1)A} = \frac{u}{u_c} \quad (20a)$$

$$C^2 = \gamma^2 \left(\frac{u}{u_c}\right)^{\frac{p}{p+1}} \exp\{\epsilon(u_c^2 - u^2)\} \quad (20b)$$

where  $\gamma = r_c + O(\epsilon)$  is of order unity and given by integrating out from the core. From these we may read off:

$$x_{CK} = \frac{e^A}{C} \left[ \epsilon u - \frac{p}{2(p+1)u} \right] \quad (21a)$$

$$y_{CK} = \frac{e^A}{C} \left[ \epsilon u + \frac{p}{2(p+1)u} \right] \quad (21b)$$

Therefore the CK trajectory crosses the  $y$ -axis at

$$y_{CK} = \frac{2}{\gamma^2} e^{-1/2\epsilon} e^{\frac{p}{4(p+1)}} \left( \frac{p\epsilon}{2(p+1)} \right)^{\frac{3p+4}{4(p+1)}} \quad (22)$$

Provided that this is greater than, or even roughly of the same order as  $pH$ , this will be a good approximation to the actual nonsingular trajectory crossing the  $y$ -axis.

Finally, note that for the actual phase plane trajectories

$$\frac{dy}{dx} = (p+1) - \frac{py((p+1)y-x)}{(p+1)y^2 + p^2H^2 - x^2} > (p+1) - \frac{py((p+1)y-x)}{(p+1)y^2 - x^2} = \frac{dy_{CK}}{dx_{CK}} \quad (23)$$

for  $y > x > 0$ . This means that the true trajectories are slightly steeper than the CK trajectories, but as already noted, the deviation is minor while  $y \geq O(pH)$ .

Now let us analyse the behaviour of the trajectory near  $P_-$ . In the vicinity this non-degenerate critical point, the dynamical system is approximately linear and given by

$$\begin{bmatrix} \frac{d\bar{x}}{d\rho} \\ \frac{d\bar{y}}{d\rho} \end{bmatrix} = H\mathbf{M} \begin{bmatrix} \bar{x} \\ \bar{y} \end{bmatrix} \quad (24a)$$

with

$$\mathbf{M} = \begin{bmatrix} -2 & 0 \\ -2(p+1) & p \end{bmatrix} \quad (25a)$$

and where  $\bar{x} = x + pH$  and  $\bar{y} = y$ , with  $|\bar{x}|, |\bar{y}| \ll pH$ . Calculating the eigenvectors relative to the eigenvalues of the matrix  $\mathbf{M}$  one finds that the nonsingular trajectory approaching  $P_-$  is

$$\bar{y} = 2 \left( \frac{p+1}{p+2} \right) \bar{x} \quad (26)$$

whereas trajectories with  $\bar{x} = 0$  repel from that point. Note that (26) leads to the asymptotic solutions given in (14a).

We would now like to estimate where this nonsingular trajectory crosses the  $y$ -axis. Consider the straight line

$$y = m(x + pH) \quad (27)$$

for  $x > -pH$ . Then, along this line we may use (10a,10b) to obtain:

$$\frac{dy}{d\rho} - m \frac{dx}{d\rho} = \frac{y}{m^2 p} \left[ (m^2 - 1)(m - 1)(p + 1)y + (m(p + 2) - 2(p + 1)) mpH \right] \quad (28)$$

Thus (remembering that both  $y$  and  $x$  are *decreasing* along trajectories) we have:

$$\frac{dy}{dx} \begin{cases} \leq m & \text{for } m \geq \frac{2(p+1)}{(p+2)} \\ > m & \text{for } m = 1. \end{cases} \quad (29)$$

This means that the trajectory passing through  $(0, \frac{2(p+1)}{(p+2)}pH)$  misses  $P_-$ , turns around and asymptotes a singular CK trajectory in the upper left hand quadrant of the phase plane. On the other hand, the trajectory passing through  $(0, pH)$  also misses  $P_-$ , but crosses



$x = 0$  passing to the lower left hand quadrant (which is also a singularity). Therefore, the nonsingular trajectory crosses the  $y$  axis in the range  $(pH, \frac{2(p+1)}{(p+2)}pH)$ . Combining this with the fact that the true trajectories are slightly steeper than the CK trajectories, means that we get a very good approximation to  $H$  by matching the CK trajectory at  $x = 0$  to the critical trajectory (26):

$$H = h(p)\epsilon^{\frac{3p+4}{4(p+1)}}e^{-1/2\epsilon} \quad (30)$$

where

$$h(p) = \frac{2(p+2)}{\gamma^2 p^2} e^{\frac{p}{4(p+1)}} \left( \frac{p}{2(p+1)} \right)^{\frac{7p+8}{4(p+1)}} \quad (31)$$

is of order  $O(1/p)$ .

Therefore, we have demonstrated the existence of a nonsingular trajectory corresponding to the exterior spacetime of a global vortex  $p$ -brane with a de-Sitter like expansion parallel to the brane with the Hubble constant of this expansion being very finely tuned to the gravitational strength of the vortex, and given by (30).

### III. GLOBAL STRUCTURE AND MAXIMAL EXTENSION

In the previous section we demonstrated the existence of a non-singular solution to the vortex equations, and derived a good approximation to the spacetime within the horizon. This spacetime has the form of (3) with a good approximation to the functions  $A$  and  $C$  being given by:

$$e^A = \begin{cases} (1 - \epsilon \ln r)^{\frac{1}{p+1}} & r < r_1 \\ H(r_H - r) & r > r_1 \end{cases} \quad (32a)$$

$$C = \begin{cases} (1 - \epsilon \ln r)^{\frac{-p}{2(p+1)}} r^{1 - \frac{\epsilon}{2} \ln r} & r < r_1 \\ C_0 \left[ 1 - \frac{\epsilon}{(p+2)C_0^2} (r_H - r)^2 \right] & r > r_1 \end{cases} \quad (32b)$$

where  $(1 - \epsilon \ln r_1) \simeq \epsilon^{1/2}$ , with  $r_H \simeq r_1 + \epsilon^{1/2(p+1)}H^{-1}$  and  $C_0 \simeq \epsilon^{1/2}H^{-1}$ .

Before deriving the maximal extension and global spacetime structure of the vortex  $p$ -brane, it is first instructive to examine de Sitter spacetime in this cylindrical coordinate system. Recall that  $n$ -dimensional de Sitter spacetime can be represented as a hyperboloid in  $(n + 1 = p + 4)$ -dimensional flat Minkowski spacetime:

$$\mathbf{X}_{p+1}^2 - T^2 + Y^2 + Z^2 = H^{-2} \quad (33)$$

where  $H$  is the Hubble constant for the de Sitter space. Conventionally, one coordinatises the hyperboloid by setting  $T = \sinh t$  etc., however, to make contact with the form of (3) instead consider the transformation

$$(T, \mathbf{X}_{p+1}) = H^{-1} \cos Hr (\sinh t, \cosh t \mathbf{n}_{p+1}) \quad (34a)$$

$$Y + iZ = H^{-1} \sin Hr e^{i\theta} \quad (34b)$$

(where  $\mathbf{n}_{p+1}$  is the unit vector in  $(p+1)$  dimensions). This gives the metric

$$ds^2 = H^{-2} \cos^2 Hr \left[ dt^2 - \cosh^2 t d\Omega_p^2 \right] - dr^2 - H^{-2} \sin^2 Hr d\theta^2 \quad (35)$$

Note how this metric satisfies the same core boundary conditions on  $A$  and  $C$  as the vortex metric, and as  $r \rightarrow \pi/2H$ , we have similar behaviour as the vortex horizon, with  $e^A \sim H(r_H - r)$ , although here  $C_0 = H^{-1}$  rather than the  $\epsilon^{1/2}H^{-1}$  of the vortex horizon.

Clearly this coordinate system does not cover the full hyperboloid as it does not allow  $Y^2 + Z^2 > H^{-2}$ . For this region we must instead take

$$(T, \mathbf{X}_{p+1}) = H^{-1} \sinh H\tau (\cosh \xi, \sinh \xi \mathbf{n}_{p+1}) \quad (36a)$$

$$Y + iZ = H^{-1} \cosh H\tau e^{i\theta} \quad (36b)$$

giving

$$ds^2 = d\tau^2 - H^{-2} \sinh^2 H\tau \left[ d\xi^2 + \sinh^2 \xi d\Omega_p^2 \right] - H^{-2} \cosh^2 H\tau d\theta^2 \quad (37)$$

as the metric in the region ‘outside’ the event horizon. We therefore see that the horizon is, as expected, simply an artifact of the choice of coordinates, which, by splitting off the  $Y$  and  $Z$  directions, take a ‘strip’ of the hyperboloid only and the actual de Sitter spacetime is of course completely regular across the horizon.

Let us now turn to the vortex metric, which, in the neighborhood of the horizon, can be easily shown to be  $(p+2)$ -dimensional Minkowski spacetime times a (rather large) circle. (Take  $T = (r_H - r) \sinh t$  and  $\mathbf{X}_{p+1} = (r_H - r) \cosh t \mathbf{n}_{p+1}$ .) We have already remarked on the similarities between this horizon and the horizon of the de Sitter spacetime in cylindrical coordinates, therefore we set

$$(T, \mathbf{X}_{p+1}) = H^{-1} e^A (\sinh t, \cosh t \mathbf{n}_{p+1}) \quad (38a)$$

$$Y + iZ = C(r) e^{i\theta} \quad (38b)$$

which satisfies

$$\mathbf{X}_{p+1}^2 - T^2 + Y^2 + Z^2 = C^2 + H^{-2} e^{2A}. \quad (39)$$

For our solution, this RHS is very nearly constant:  $C^2 + H^{-2} e^{2A} \simeq H^{-2}(1 + O(\epsilon))$ , and in fact even for the rather large value of  $\epsilon = 0.1$ , changes by only 1 part in a billion. This means that topologically, within its event horizon at least, the vortex spacetime looks to be the same as de Sitter with spherical spatial topology. Metrically, the induced metric on the vortex hyperboloid is

$$ds_{\text{ind}}^2 = H^{-2} e^{2A} \left[ dt^2 - \cosh^2 t d\Omega_p^2 \right] - C^2 d\theta^2 - (A'^2 e^{2A} H^{-2} + C'^2) dr^2 \quad (40)$$

Because of the presence of  $H^{-2}$ , this appears at first sight to be different to (3), but in fact this term is only significantly different from 1 in the CK régime,  $r < r_1$ . Here  $C' \sim 1 + O(\epsilon)$ , and since

$$A'^2 e^{2A} H^{-2} dr^2 = \frac{(\mathbf{X} \cdot d\mathbf{X} - T dT)^2}{\mathbf{X}^2 - T^2} = H e^{-A} \frac{(\mathbf{X} \cdot d\mathbf{X} - T dT)^2}{|\mathbf{X}^2 - T^2|^{1/2}} \quad (41)$$

(dropping the subscript  $(p+1)$  on  $\mathbf{X}$  for convenience) we can obtain (3) by making a small distortion of the flat metric of the spacetime in which the vortex-hyperboloid lives.

We now need to consider the analytic extension of the vortex metric across the horizon. Note that the vortex fields inside the CEH depend on  $r$ , which is given implicitly via

$$\mathbf{X}_{p+1}^2 - T^2 = H^{-2} e^{2A} \quad (42)$$

*i.e.*,  $X$ ,  $A$ , and  $C$  are constant on spacelike hyperboloids in  $(T, \mathbf{X})$ -spacetime. In this interpretation, the CEH is the lightcone centered on the origin. Therefore, the analytic continuation across the CEH would naturally correspond to the interior of the future (or past) lightcone of the origin, which means we need the vortex and metric functions to depend on  $T^2 - \mathbf{X}^2$  which is a timelike coordinate. Note that this is *not* the way Nogales and Wang tried to continue across the CEH in [12], they attempted to keep all fields depending on  $r$  and took  $r > r_H$ . However, the coordinate transformation in the neighborhood of the CEH shows clearly that  $r > r_H$  corresponds to  $z \rightarrow z + \pi$  ( $z$  being the coordinate along the 1-dimensional vortex), hence this identifies antipodal points on the CEH which renders the spacetime singular. It is likely that this was the singularity they were finding evidence of, for as we shall see, the CEH is simply a coordinate singularity, and does not appear to be a future Cauchy horizon, therefore one would not expect any instabilities there.

Using the de Sitter spacetime as a guide, we therefore look for a time dependent solution of the Goldstone model (with topological winding in the scalar field) by replacing (3) with

$$ds^2 = d\tau^2 - H^{-2} e^{2A(\tau)} \left[ d\xi^2 + \sinh^2 \xi d\Omega_p^2 \right] - C^2(\tau) d\theta^2 \quad (43)$$

The Einstein equations for this metric, (setting  $\Phi = \eta e^{i\theta}$ ) give

$$\frac{d}{d\tau} \left[ \dot{C} e^{(p+1)A} \right] = \frac{2\epsilon e^{(p+1)A}}{C} \quad (44a)$$

$$\frac{d}{d\tau} \left[ C e^{(p+1)A} \dot{A} \right] = p H^2 C e^{(p-1)A} \quad (44b)$$

$$\frac{p}{2} (H^2 e^{-2A} - \dot{A}^2) - \frac{\dot{A}\dot{C}}{C} = -\frac{\epsilon}{(p+1)C^2} \quad (44c)$$

which are the same as (4a-4c) with  $\epsilon \rightarrow -\epsilon$ .

As before, by writing  $\rho = -\int e^{-A} dt$ , we see that these equations of motion give the same two dimensional dynamical system (10a,10b), but with the constraint (11) now reading

$$\frac{2\epsilon p e^{2A}}{(p+1)C^2} = (x^2 - y^2 - p^2 H^2) . \quad (45)$$

Note the minus sign in the definition of  $\rho$ : this is so that the event horizon corresponds to the critical point

$$x = p \frac{dA}{d\rho} + \frac{1}{C} \frac{dC}{d\rho} = -e^A \left[ p \frac{dA}{d\tau} + \frac{dC}{d\tau} \right] = -pH \quad (46)$$

as  $e^A \sim H\tau$  near the horizon.

In other words, whereas the vortex dynamical system corresponded to the central region in the phase plane in between the two branches of the invariant hyperboloid, the spacetime ‘exterior’ to the horizon corresponds to the disconnected regions outside the branches of the invariant hyperboloid. Indeed, since we know the event horizon corresponds to the critical point  $P_-$  on the left branch of the invariant hyperboloid, we conclude that the spacetime exterior to the event horizon corresponds to the left part of this phase plane,  $y^2 < x^2 - p^2 H^2$ .

The phase plane trajectories on the left of the invariant hyperboloid generically emerge from the focus (or repeller, depending on the value of  $p$ )  $Q_-$ , and are attracted to the branch of the invariant hyperboloid in the upper left hand part of the phase plane, *i.e.*, the Cohen-Kaplan solution which is singular. There is however one trajectory which terminates on the critical point  $P_-$ , and this is clearly the trajectory corresponding to the exterior-horizon solution. We can therefore read off the late time solution from (14b) as

$$ds^2 = d\tau^2 - \frac{p\tau^2}{(p+1)} \left[ d\xi^2 + \sinh^2 \xi d\Omega_p^2 \right] - \frac{2\epsilon}{(p+1)} \tau^2 d\theta^2. \quad (47)$$

This is a late time inhomogeneous open universe cosmology.

Finally, defining

$$T = H^{-1} e^A \cosh \xi \quad (48a)$$

$$\mathbf{X}_{p+1} = H^{-1} e^A \sinh \xi \mathbf{n}_{p+1} \quad (48b)$$

$$Z + iY = C(\tau) e^{i\theta} \quad (48c)$$

now gives

$$ds^2 = d\tau^2 \left[ H^{-2} \dot{A}^2 e^{2A} - \dot{C}^2 \right] - H^{-2} e^{2A} \left[ d\xi^2 + \sinh^2 \xi d\Omega_p^2 \right] - C^2 d\theta^2 \quad (49)$$

Using the phase plane trajectory of the solution, we can see that

$$\left[ H^{-2} \dot{A}^2 e^{2A} - \dot{C}^2 \right] = \frac{(x-y)^2}{p^2 H^2} - \frac{2\epsilon p y^2}{(p+1)(x^2 - y^2 - p^2 H^2)} \quad (50)$$

can be bounded by

$$\left( 1 - \frac{1}{\sqrt{p(p+1)}} \right)^2 - O(\epsilon) < \left[ H^{-2} \dot{A}^2 e^{2A} - \dot{C}^2 \right] < \frac{p+1}{p} \quad (51)$$

hence (49) is indeed a good approximation to the true exterior metric (43).

Note however that as a surface in Minkowski  $\mathbb{R}^{p+4}$ , the vortex departs from the deformed de Sitter-type hyperboloid form that it had within the event horizon, as (48a-48c) give

$$2\epsilon(T^2 - \mathbf{X}_{p+1}^2) - p(Y^2 + Z^2) = \frac{2\epsilon e^{2A}}{H^2} - pC^2 \simeq 0 \quad (52)$$

at late times, which is a timelike hyperboloid, and hence spacetime is asymptotically flat here as required by consistency with (47). A sketch of the vortex hyperboloid in  $\mathbb{R}^{p+4}$  is shown in figure 2.

To summarize: we have shown that the vortex (like the domain wall) compactifies space on a scale roughly of order  $O(H^{-1})$ . A  $t = \text{constant}$  section has the topology of a  $(p+1)$ -sphere, in which the vortex could be thought of as sitting on a great circle, however, as the  $S^{p+1}$  is deformed, this picture should be treated with caution.

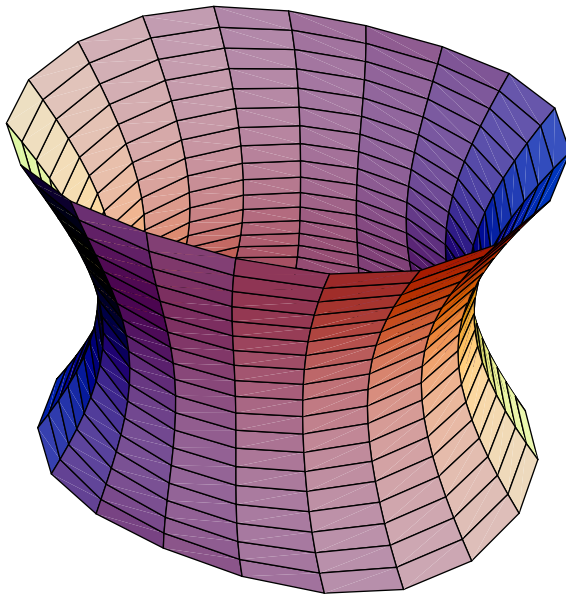


FIG. 2. The deformed hyperboloid of the vortex spacetime. The left and right side are close to a genuine hyperboloid, and correspond to the region inside the CEH. The ‘front’ and ‘back’ however are flattened, and correspond to the hyperboloid straightening out to asymptote the flat spacetime outside the CEH.

#### IV. STRONGLY GRAVITATING VORTICES

An interesting question we can ask is what happens for more strongly gravitating vortices, *i.e.*, for  $\epsilon \simeq 1$ . As  $\epsilon$  becomes larger, the spacetime reacts more and more strongly to the vortex, and the field theory solution departs more and more from the flat space vortex. Meanwhile,  $H$  is increasing, and the CEH correspondingly moves inwards rather rapidly leaving less distance outside the vortex core. At some point it may well be that the CEH would like to be *inside* the vortex core. If this occurs, then the vortex fields are essentially in their false vacuum state and one might expect that the ‘vortex’ as such disappears and the solution becomes de Sitter, the false vacuum energy of the unbroken vacuum providing the cosmological constant. This is in fact the set-up of the topological inflation model of Linde and Vilenkin [19].

In the notionally similar case of the domain wall, it is known that in the absence of gravitational back reaction, a kink/antikink pair can be solved exactly on a compact  $S^1$  in terms of elliptic functions [20]. It turns out that there is a critical radius for the circle below which the kink/antikink pair cannot exist and the only solution is either the false vacuum, or one of the true vacuum choices. Once gravitational back reaction is correctly taken account of [10] this qualitative result also holds. The gravitational back reaction of the wall introduces a compactification of space at a scale of the inverse gravitational coupling ( $\epsilon$ ). As  $\epsilon$  grows, this compactification radius shrinks and at some critical gravitational strength,

there is no longer enough space for the kink to exist and the only possible solution is the false vacuum (which gives a de Sitter spacetime) or the true vacuum.

Here for the global vortex  $p$ -brane we no longer have explicit solutions<sup>1</sup>, moreover,  $H$  is determined in a rather indirect way, therefore we do not have as direct an intuition on what happens as  $\epsilon$  increases, nonetheless, in a manner similar to the arguments of [10], we can in fact give analytic arguments as to the typical value of  $\epsilon$  at which the vortex solution will cease to exist and the only possibilities will be the true vacuum or topological inflation.

There are two ways of limiting the value of  $\epsilon$  at which the vortex ceases to exist. Note that the false vacuum de Sitter (FVDS) solution,  $X \equiv 1$  and metric (35) with  $H^2 = \epsilon/2(p+1)(p+2)$ , is always a solution to the equations of motion (4). The first method, which gives a lower bound to  $\epsilon_c$ , is to ask at what value of  $\epsilon$  this FVDS solution becomes unstable to vortex formation. Clearly, if the FVDS solution is unstable to vortex formation, we are below the critical value of  $\epsilon$ . The second method looks at the vortex field equation (4d). If a vortex spacetime exists, then there will be a solution to this equation (along with the accompanying  $A$  and  $C$ ) which starts with  $X = 0$  at  $r = 0$ , and goes to  $X = X_H$  at  $r = r_H$ , with  $X'(r_H) = 0$  required by nonsingularity of the spacetime. This places some demands on the behaviour of the function  $X$  which cannot be satisfied if  $\epsilon$  is too large. This obviously gives an upper bound on  $\epsilon_c$ .

For the first method we need to analyze the perturbation equations of the FVDS solution:

$$\square\xi + \frac{\xi}{C^2} - \frac{\xi}{2} = 0 \quad (53)$$

It is most transparent to use planar coordinates in the vortex worldvolume in (3), *i.e.*,  $H^{-2}g_{\mu\nu}dx^\mu dx^\nu = dt^2 - e^{2Ht}dx_p^2$  in which this perturbation equation takes the form

$$\left[\ddot{\xi} + pH\dot{\xi}\right]\sec^2 Hr - \xi'' - (H \cot Hr - (p+1)H \tan Hr)\xi' + (H^2 \csc^2 Hr - \tfrac{1}{2})\xi = 0 \quad (54)$$

This has solution

$$\xi = e^{\nu Ht} \sin Hr \cos^\nu Hr \quad (55)$$

where

$$\nu = -\frac{(p+4)}{2} \pm \frac{1}{2}\sqrt{(p+2)^2 + \frac{4}{\epsilon}(p+1)(p+2)} \quad (56)$$

For an instability, we require a perturbation which grows with time, *i.e.*,  $\nu > 0$ . Clearly this requires

$$\epsilon < \frac{(p+1)(p+2)}{(p+3)} \leq \epsilon_c \quad (57)$$

This gives the lower bound for the critical value  $\epsilon_c$ .

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<sup>1</sup>Local vortices on compact spaces have been studied [21], however to our knowledge, global vortices on spheres have not.

For the second method, we now try to look for a vortex-type solution, with  $X'(0) > 0$  and  $X'$  tending monotonically to zero at the horizon (we can show this from the equations of motion, but let us simply state it as a property a vortex solution would display). Then, an examination of the equations of motion (4) shows that  $e^A$  must be monotonically decreasing from  $r = 0$  to  $r_H$ , and  $X'(r_H) = 0$ . Meanwhile, the value of  $H$  must be less than the pure FVDS solution,  $H^2 \leq \epsilon/2(p+1)(p+2)$ .

Taylor expanding the functions around  $r = 0$  ( $X = x_1 r + x_3 r^3/6$  etc.) and using (4) gives

$$(p+1)a_2 = -\frac{\epsilon}{4} + \frac{p(p+1)H^2}{2} < -\frac{\epsilon}{2(p+2)} \quad (58a)$$

$$c_3 = -2\epsilon x_1^2 - \frac{\epsilon}{2(p+1)} - (p+1)a_2 \quad (58b)$$

$$x_3 = \frac{x_1}{4} \left( 4\epsilon x_1^2 + \frac{\epsilon}{(p+1)} - \frac{3}{2} - (p+1)a_2 \right) \quad (58c)$$

We can therefore deduce that if  $\epsilon > 3(p+1)/2$ ,  $x_3 > 0$  and  $2x_3 + c_3 x_1 > 0$ . This means that in a neighborhood of the origin,  $X''$  and  $F = X' - X/C$  are positive.

Now let us examine what happens as we move towards the horizon. Since  $F(r_H) < 0$ ,  $F$  must have a zero on  $(0, r_H)$  with  $F' < 0$  at this point. However,

$$F' = \left( \frac{C}{2} - \frac{C'}{C} - \frac{1}{C} \right) F + \frac{X^3}{2} + X' \left[ -(p+1)A' - \frac{1}{2}C \right] > X'G|_{F=0} \quad (59)$$

where  $G = \left[ -(p+1)A' - \frac{1}{2}C \right]$ . However, from (58) we see that  $G > 0$  near the origin, and indeed

$$G' = (p+1)A'^2 + \epsilon F \left( X' + \frac{X}{C} \right) + \frac{C'}{C}G \quad (60)$$

is clearly positive while  $F$  and  $G$  are both positive. Thus  $G > 0$  while  $F > 0$ , hence  $F$  cannot have a zero from (59).

We conclude that for  $\epsilon > 3(p+1)/2$  we cannot have a vortex-type solution. The critical value of  $\epsilon$  therefore lies in the range

$$\frac{(p+1)(p+2)}{(p+3)} \leq \epsilon_c \leq \frac{3(p+1)}{2} \quad (61)$$

Since  $p \geq 1$ , we see that the value of  $\epsilon$  for which the vortex solution ceases to exist is actually quite high,  $\epsilon_c \geq \frac{3}{2}$  and grows linearly with  $p$ . We expect that the CEH in these cases actually occurs for quite low values of  $X_H$ .

## V. DISCUSSION

Since we have shown how the vortex compactifies space on a scale of order  $H^{-1}$ , a natural question to ask is how this feeds back into a possible braneworld type of resolution of the hierarchy problem. Briefly, the braneworld picture (pioneered by Rubakov, Shaposhnikov

and Akama [22]) imagines that our universe is simply a defect or submanifold embedded within a higher dimensional manifold. We are confined to live on this submanifold, but gravity can propagate throughout the full dimensionality of spacetime. Such models can either have a Kaluza-Klein type of picture of gravity [23], or can limit long range effects of the extra dimensions via warped compactifications (e.g. [13]). The way such models provide an explanation of the hierarchy between particle and gravitational interaction is via the geometrical volume of the transverse space (or a factor similar to this for the warped compactifications).

The hierarchy generation by a vortex compactification was first explored by Cohen and Kaplan [16] for their singular exact solution. In fact, if we suppose there is no bulk cosmological constant (as CK did) but that we have our worldvolume Hubble expansion, then we can quite straightforwardly derive the hierarchy factor in our case, for writing the metric in the form

$$ds^2 = e^{2A} \tilde{g}_{\mu\nu}(x) dx^\mu dx^\nu - dr^2 - C^2 d\theta^2 \quad (62)$$

gives

$$M_n^{p+1} \int \sqrt{g_n} R_n d^n x \sim 2\pi M_n^{p+1} \int_0^{r_H} C e^{(p-1)A} dr \int \tilde{R} \sqrt{\tilde{g}} d^{p+1} x \quad (63)$$

which, upon integration of (4b) from  $r = 0$  to  $r_H$  gives

$$M_{(p+1)}^{p-1} \simeq \frac{\pi \epsilon M_n^{p+1}}{p(p+1)H^2} \quad \Rightarrow \quad M_{Pl}^2 = \frac{\pi \eta^2}{24H^2} \quad (64)$$

for a vortex compactification of six to four dimensions. Because of the exponential dependence of  $H$  on  $\epsilon$ , it is not difficult to get a large hierarchy between the six and four dimensional Planck masses. Putting  $p = 3$  in (30) and (31), and recalling that in vortex units  $M_n \simeq \epsilon^{-1/(p+1)}$ , gives

$$\frac{1}{\epsilon} - \frac{9}{8} \ln \epsilon = 4.6 \left( 16 - \ln \left( \frac{M_n}{1 \text{ TeV}} \right) \right) \quad (65)$$

For a six dimensional Planck scale of 10 TeV, this gives the vortex scale of around 3.6 TeV, for  $M_6 \simeq 100 \text{ TeV}$ , the vortex scale is around 37 TeV.

One interesting feature of this type of compactification is that the vortex worldvolume is not Minkowski flat spacetime, but rather an inflating de Sitter universe. Since we suspect that our universe is in fact asymptoting a mild de Sitter universe [24], it is tempting to use this Hubble expansion to provide the  $\Omega_\Lambda \simeq 0.7$  that we see today. Unfortunately, the Hubble parameter provided by (30), while ‘small’ in vortex units, is simply too large to provide the tiny  $\Omega_\Lambda$  we see today. However, if we try to construct an admixture of the RS-style warped compactification of [17] and the Hubble expansion of the pure vacuum vortex, we can obtain a more general solution.

Technically, the addition of a cosmological constant makes the dynamical system phase space three dimensional, however, the CEH remains a critical point, and now *two* of the three eigenvectors are attractive. This means that there is a two parameter family of solutions approaching this point, therefore given the initial conditions outside the core of the vortex,



we are still guaranteed to be able to match this trajectory to one terminating on the critical point by varying  $H$ . We have shown in this paper how one can have a nonsingular solution with no  $\Lambda$  but with a worldvolume Hubble expansion,  $H$ . In [17] it was shown a nonsingular solution was possible with negative  $\Lambda$  but no  $H$ . Roughly speaking we can therefore find a whole family of solutions with a cosmological constant and Hubble expansion  $H(\Lambda)$  where  $H'(\Lambda) > 0$ . In principle therefore, by simply de-tuning the flat Randall-Sundrum type compactification of [17], we can construct a global vortex compactification that both solves the hierarchy problem as well as giving us the required  $\Omega_\Lambda$  we see today. Unfortunately, as there is nothing either ‘natural’ or generic about this de-tuning, this ‘resolution’ of the cosmological constant problem is only artificial, in that it simply sweeps the problem under the rug of higher dimensions.

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